**Project description:**

For this project, I plan to forecast the turnover of household goods retailing. The data series is included in the Retail Trade, Australia (code: 8501.0) with a Series ID A3348600A.

Household goods retailing is a significant component of Australian Retail Trade. The change of its turnover can intuitively reflect the demand in the household retailing market, outlook of Australian Retail Trade, and consumption level. In addition, Retail Trade has been a significant part for contributing Australia’s economy for a long period. Observing the household retailing past turnover will not only helps us make valuable forecast but also gives us hints about economy’s trends and cycles. This data series is published by Australian Bureau of Statistics (ABS) monthly since 1982, it can be obtained from the data downloads table 1 on the ABS website:

<https://www.abs.gov.au/statistics/industry/retail-and-wholesale-trade/retail-trade-australia/latest-release#data-download>

The upcoming release for Retail Trade, Australia is 4th November 2020.

**Personal Project Report:**

**Data series:**

**Growth Rate:**

Based on the graph of original data, we could see a strong and clear growth rate in the figure, as the household retailing keeps going up, the fluctuation in middle and end periods gets much bigger but the steady upward trend is shown. It is plausibly for us to say there is a positive linear or exponential relationship between the percentage Household retailing and time series.

**Brief analysis of trend:**

The increasing trend in the beginning part of the Household Goods Retailing figure is not very strong, it takes almost 200 periods to achieve the Turnover of 2000. However, after that, the increasing trend is much stronger, which can be explained as the rapid development of productivity and the improvement of people’s consumption level from rapid increasing employment population.

**Seasonality:**

According to the auto-covariance figure and based on the original figure, it looks like Household Goods Retail is significantly influenced by some level of seasonality.

**Description of forecasting methods:**

I use the random walk as the benchmark, then I considered OLS with seasonal dummies; based on experience and the good performance of model specifications, I also use the explanatory variables with seasonal dummies and the Holt-Winter model. The trend cycle model is used as well, it is just for testing whether it would be better than previous specifications.

Next, since the auto-covariance for Household Retail shows data series may be the AR dominant models, then I try AR (p), IAR(p) and ARMA (p, q). Assuming AR error term follows white noise, ARMA (p, q) error term follows moving average. The performance of them is generally worse than OLS with seasonal dummies, especially for ARMA (p, q) with very large MSFE compared to random walk, the moving average assumption could not be hold based on the performance, then I choose to use the for AR (p) and ARMA (p, q). Besides, the unobserved component model is just used for finding whether there would be better forecast models as well.

图表, 折线图, 直方图

描述已自动生成图表, 直方图

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Original series Auto-covariance for original series

**Evaluation methods**

For the efficacy of forecast, I choose to use MSFE one and three steps ahead forecast. When comparing different models and specifications in the project, I always choose the one with lower MSFEs. All T0 = 50 in this project for the consistency of forecast results.

**Model specification**

**Benchmark**

Use the random walk as my benchmark: ; the forecast value is .

|  |  |
| --- | --- |
| MSFE 1-step ahead | MSFE 3-step ahead |
| 233545.233260341 | 283320.598557457 |

图表, 直方图

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Benchmark 1-step ahead Benchmark 3-step ahead

**Seasonal dummy**

The auto-covariance and the figure of Household Goods Retailing give us the intuition that the data series have the seasonality. There are four potential specifications as shown in below; For S1, the model has a start value, a trend and one fourth quarter dummy; S2 has a trend component and 4 quarterly dummies; S3 has a star value, a trend and one Dec dummy; S4 has a trend and 12 dummies for each month. The date was changed to repeat number sequences 1,1,1, 2,2,2,3,3,3,4,4,4 for adding seasonal dummies, then put repeating number sequences 1 to 12 for monthly dummies in the column C.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | S1 | S2 | S3 | S4 |
| AIC | 64300303.4121504 | 59518468.5672141 | 63409658.1266137 | 30069933.6426960 |
| BIC | 64300315.8123446 | 59518489.2342043 | 63409670.5268078 | 30069987.3768705 |
| MSE | 139480.037770391 | 129107.285395258 | 137548.052335388 | 65227.5653854576 |
| MSFE 1-step | 156642.044701318 | 146360.830606889 | 154236.723863778 | 76615.7814139809 |
| MSFE 3-steps | 160087.162576580 | 149646.501509459 | 157937.906444184 | 79185.9422993936 |

图表, 直方图

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S1 1-step ahead S1 3-steps ahead

图表, 折线图, 直方图

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S2 1-step ahead S2 3-steps ahead

图表, 折线图, 直方图

描述已自动生成图表, 折线图, 直方图

描述已自动生成

S3 1-step ahead S3 3-steps ahead

图表, 直方图

描述已自动生成图表, 直方图

描述已自动生成  
 S4 1-step ahead S4 3-steps ahead

Household Retail has a close relationship with employment, in this case, for making better forecast, I add Employed total persons (Mar 1982 – July 2020 for MSFE 1, Jan 1982-May 2020 for MSFE3, code:6291.055.001) as a predictor for improving forecasts.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| MSFE1 | 143641.326206138 | 133859.777311577 | 141522.078556514 | 63489.6670227383 |
| MSFE3 | 144133.150536213 | 132262.071569209 | 139602.666549890 | 65893.8723615432 |

图表, 折线图, 直方图

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描述已自动生成

S’1 1-step ahead S’1 3-steps ahead

图表, 折线图, 直方图

描述已自动生成图表, 折线图, 直方图

描述已自动生成

S’2 1-step ahead S’2 3-steps ahead

图表, 折线图, 直方图

描述已自动生成图表, 直方图

描述已自动生成

S’3 1-step ahead S’3 3-steps ahead

图表, 折线图, 直方图

描述已自动生成图表, 折线图, 直方图

描述已自动生成

S’4 1-step ahead S’4 3-steps ahead

**Holt-Winter**

I consider use the Holt-Winter smoothing for getting possible lower MSFE. In order to minimize the MSFE, I find the optimal parameters. The Holt-Winter illustrates a significant improvement for both MSFE 1-step and 3-step ahead forecast.

In this situation, MATLAB shows the MaxFunEvals issue, then I divided all data by 100 to get the optimal parameters, I don’t know the exact reason, but it works. After that, I changed the data back for further calculation. It gives alpha = 0.6660, beta = 0.0589, gamma = 0.9215 for MSFE one step ahead; alpha = 0.0000, beta = 3.6248, gamma = 0.0730.

|  |  |
| --- | --- |
| MSFE 1-step | MSFE 3-step |
| 9914.93303690565 | 1374151.00018113 |

**Trend Cycle Model**

|  |  |
| --- | --- |
| MSFE 1-step | 117159.975809856 |

**AR(p)**

I consider use auto-regressive process from AR (1) to AR (4),

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | AR (1) | AR (2) | AR (3) | AR (4) |
| MSFE 1-step | 152865.390521171 | 148329.916009018 | 138402.188331872 | 137397.802237105 |

AR ( )

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | AR (1) | AR (2) | AR (3) | AR (4) |
| MSFE 1-step | 146568.360924178 | 136453.725293962 | 135564.728015979 | 122336.857020005 |

(I put in column 6 and the make the first one equal to the second one, just change the y = Household (:, 2) to y = Household (:, 6) will give these results)

**IAR (p)**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | IAR (1) | IAR (2) | IAR (3) | IAR (4) |
| MSFE 1-step | 153633.809420899 | 148713.697330458 | 138402.188331872 | 137079.663063778 |

**ARMA (p, q)**

The auto-covariance plot given in the beginning shows the AR dominance and the error term may follow the moving average. Therefore, there are potential specifications of ARMA (p, q).

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | ARMA (1,1) | ARMA (2,1) | ARMA (3,1) | ARMA (4,1) |
| MSFE 1-step | 8.29325095776412e+149 | 2.98611166288720e+192 | Inf | Inf |

(With MA in models, MSFEs look terrible and even have infinite value by computer’s calculation.)

ARMA (p, q) with

|  |  |  |
| --- | --- | --- |
|  | ARMA (1,1) | ARMA (2,1) |
| MSFE 1-step | 1.30291287292052e+151 | Inf |

**Unobserved Component model**

|  |  |
| --- | --- |
| MSFE 1-step | 8855589.19720066 |

**Conclusion**

1. The data series have a relatively clear trend and seasonality, which reflects into the MSFE from models.
2. The explanatory variables of employment improve the forecast a lot, which gives the intuition about adding more potential variables could make more accurate forecast.
3. The data series looks like auto-regressive and these AR and IAR models truly give us the better MSFE compared to random walk.
4. Moving average assumption is terrible, it destroys models and makes very large MSFEs, which obviously should not be considered into any model for original data.
5. Using first difference helps us to get better MSFEs for AR but those are still larger than previous model forecasts. However, it is much worse when using the ARMA models, which gives me the most reason for stopping using MA in the forecast.
6. Trend cycle model is not a very bad model for forecasting, which even performs better than AR(p); but the unobserved component model does not perform well.
7. Overall, the Holt-winter, S4 and S’4 are best forecast specifications.
8. Density Forecast:

|  |  |  |
| --- | --- | --- |
|  | Holt-Winter | S’4 |
|  | 5836.57517774904 | 4429.12168088316 |
|  | 8897.46174272537 | 53720.5095285461 |
| CI | (5651.69551680481,6021.45483869328) | (3974.83904924856,4883.40431251777) |
| Average | 5132.848 | |

1. Although Holt-Winter gives the smallest MSFE, this model has hysteresis, which may result in not very precise forecast when seasonality starts to influence the data series. Considering those two models together may give a much accurate forecast.

**Code Appendix:**

**Original Data series figure and autocovariance( put time series in column 7)**

%original data series figure:

load 'Household.csv';

x=Household(:,7);

y=Household(:,2);

plot(x,y);

title('Houshold Goods Retailing')

xlabel('periods');

ylabel('Turnover')

%% Auto-covariance

[c,lags]=xcov(x);

stem(lags,c);

**PJbenchmark:**

load 'Household.csv';

y = Household(:,2); Q = Household(:,1);

T = length(y); t = (1:T)';a=1;

T0 = 50;

h = 1; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yhat1 = a\*y(t-1);

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

H = 3;

syHat = zeros(T-H-T0+1,1);

ytpH = y(T0+H:end); % observed y\_{t+h}

for t = T0:T-H

yhat2 = a\*y(t-1);

syHat(t-T0+1) = yhat2;

end

MSFE2 = mean((ytpH-syHat).^2);

b=1:461;

c=51:461;

plot(b,y);

hold on

plot(c,syhat);

hold off

m=53:461;

plot(b,y);

hold on

plot(m,syHat);

hold off

**PJS1:**

load 'Household.csv';

y = Household(:,2); Q = Household(:,1);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 4 dummy variables

D1 = (Q == 1); D2 = (Q == 2);

D3 = (Q == 3); D4 = (Q == 4);

X=[a t D4];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

MSE=mean((y-yhat).^2);

AIC=T\*MSE+3\*2;

BIC=T\*MSE+3\*log(T);

T0 = 50;

h = 1; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

Xt = [ones(t,1) (1:t)' D4t];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [1 t+h D4(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

H = 3;

syHat = zeros(T-H-T0+1,1);

ytpH = y(T0+H:end); % observed y\_{t+h}

for t = T0:T-H

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

Xt = [ones(t,1) (1:t)' D4t];

beta2 = (Xt'\*Xt)\(Xt'\*yt);

yhat2 = [1 t+H D4(t+H)]\*beta2;

syHat(t-T0+1) = yhat2;

end

MSFE2 = mean((ytpH-syHat).^2);

b=1:461;

c=51:461;

plot(b,y);

hold on

plot(c,syhat);

hold off

m=53:461;

plot(b,y);

hold on

plot(m,syHat);

hold off

**PJS1M:**

load 'Household.csv';

y = Household(:,2); Q = Household(:,1);x=Household(:,4);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 4 dummy variables

D1 = (Q == 1); D2 = (Q == 2);

D3 = (Q == 3); D4 = (Q == 4);

X=[a t D4 x];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

T0 = 50;

h = 1; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

xt = x(1:t);

Xt = [ones(t,1) (1:t)' D4t xt];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [1 t+h D4(t+h) x(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

H = 3;

syHat = zeros(T-H-T0+1,1);

ytpH = y(T0+H:end); % observed y\_{t+h}

for t = T0:T-H

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

Xt = [ones(t,1) (1:t)' D4t];

beta2 = (Xt'\*Xt)\(Xt'\*yt);

yhat2 = [1 t+H D4(t+H)]\*beta2;

syHat(t-T0+1) = yhat2;

end

MSFE2 = mean((ytpH-syHat).^2);

b=1:461;

c=51:461;

plot(b,y);

hold on

plot(c,syhat);

hold off

**PJS1M3:**

load 'Household.csv';

y = Household(:,2); Q = Household(:,1);x=Household(:,5);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 4 dummy variables

D1 = (Q == 1); D2 = (Q == 2);

D3 = (Q == 3); D4 = (Q == 4);

X=[a t D4 x];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

T0 = 50;

h = 3; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

xt = x(1:t);

Xt = [ones(t,1) (1:t)' D4t xt];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [1 t+h D4(t+h) x(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

b=1:461;

m=53:461;

plot(b,y);

hold on

plot(m,syhat);

hold off

**PJS2:**

load 'Household.csv';

y = Household(:,2); Q = Household(:,1);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 4 dummy variables

D1 = (Q == 1); D2 = (Q == 2);

D3 = (Q == 3); D4 = (Q == 4);

X=[t D1 D2 D3 D4];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

MSE=mean((y-yhat).^2);

AIC=T\*MSE+5\*2;

BIC=T\*MSE+5\*log(T);

T0 = 50;

h = 1; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

Xt = [(1:t)' D1t D2t D3t D4t];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [t+h D1(t+h) D2(t+h) D3(t+h) D4(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

H = 3; % h-step-ahead forecast

syHat = zeros(T-H-T0+1,1);

ytpH = y(T0+H:end); % observed y\_{t+h}

for t = T0:T-H

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

Xt = [(1:t)' D1t D2t D3t D4t];

beta2 = (Xt'\*Xt)\(Xt'\*yt);

yhat2 = [t+H D1(t+H) D2(t+H) D3(t+H) D4(t+H)]\*beta2;

syHat(t-T0+1) = yhat2;

end

MSFE2 = mean((ytpH-syHat).^2);

b=1:461;

c=51:461;

plot(b,y);

hold on

plot(c,syhat);

hold off

m=53:461;

plot(b,y);

hold on

plot(m,syHat);

hold off

**PJS2M:**

load 'Household.csv';

y = Household(:,2); Q = Household(:,1);x=Household(:,4);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 4 dummy variables

D1 = (Q == 1); D2 = (Q == 2);

D3 = (Q == 3); D4 = (Q == 4);

X=[t D1 D2 D3 D4 x];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

T0 = 50;

h = 1; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

xt = x(1:t);

Xt = [(1:t)' D1t D2t D3t D4t xt];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [t+h D1(t+h) D2(t+h) D3(t+h) D4(t+h) x(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

b=1:461;

c=51:461;

plot(b,y);

hold on

plot(c,syhat);

hold off

**PJS2M3:**

load 'Household.csv';

y = Household(:,2); Q = Household(:,1);x=Household(:,5);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 4 dummy variables

D1 = (Q == 1); D2 = (Q == 2);

D3 = (Q == 3); D4 = (Q == 4);

X=[t D1 D2 D3 D4 x];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

T0 = 50;

h = 3; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

xt = x(1:t);

Xt = [(1:t)' D1t D2t D3t D4t xt];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [t+h D1(t+h) D2(t+h) D3(t+h) D4(t+h) x(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

b=1:461;

m=53:461;

plot(b,y);

hold on

plot(m,syhat);

hold off

**PJS3:**

load 'Household.csv';

y = Household(:,2); M = Household (:,3);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 12 dummy variables

D1 = (M == 1); D2 = (M == 2);

D3 = (M == 3); D4 = (M == 4);

D5 = (M == 5); D6 = (M == 6);

D7 = (M == 7); D8 = (M == 8);

D9 = (M == 9); D10 = (M == 10);

D11 = (M == 11); D12 = (M == 12);

X=[a t D12];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

MSE=mean((y-yhat).^2);

AIC=T\*MSE+3\*2;

BIC=T\*MSE+3\*log(T);

T0 = 50;

h = 1; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

D5t = D5(1:t); D6t = D6(1:t);

D7t = D7(1:t); D8t = D8(1:t);

D9t = D9(1:t); D10t = D10(1:t);

D11t = D11(1:t); D12t = D12(1:t);

Xt = [ ones(t,1) (1:t)' D12t];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [1 t+h D12(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

H=3;

syHat = zeros(T-H-T0+1,1);

ytpH = y(T0+H:end);

for t = T0:T-H

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

D5t = D5(1:t); D6t = D6(1:t);

D7t = D7(1:t); D8t = D8(1:t);

D9t = D9(1:t); D10t = D10(1:t);

D11t = D11(1:t); D12t = D12(1:t);

Xt = [ ones(t,1) (1:t)' D12t];

beta2 = (Xt'\*Xt)\(Xt'\*yt);

yhat2 = [1 t+H D12(t+H)]\*beta2;

syHat(t-T0+1) = yhat2;

end

MSFE2 = mean((ytpH-syHat).^2);

b=1:461;

c=51:461;

plot(b,y);

hold on

plot(c,syhat);

hold off

m=53:461;

plot(b,y);

hold on

plot(m,syHat);

hold off

**PJS3M:**

load 'Household.csv';

y = Household(:,2);M = Household (:,3);x=Household(:,4);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 12 dummy variables

D1 = (M == 1); D2 = (M == 2);

D3 = (M == 3); D4 = (M == 4);

D5 = (M == 5); D6 = (M == 6);

D7 = (M == 7); D8 = (M == 8);

D9 = (M == 9); D10 = (M == 10);

D11 = (M == 11); D12 = (M == 12);

X=[a t D12 x];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

T0 = 50;

h = 1; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

D5t = D5(1:t); D6t = D6(1:t);

D7t = D7(1:t); D8t = D8(1:t);

D9t = D9(1:t); D10t = D10(1:t);

D11t = D11(1:t); D12t = D12(1:t);

xt = x(1:t);

Xt = [ ones(t,1) (1:t)' D12t xt];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [1 t+h D12(t+h) x(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

b=1:461;

m=51:461;

plot(b,y);

hold on

plot(m,syhat);

hold off

**PJS3M3:**

load 'Household.csv';

y = Household(:,2); M = Household (:,3);x=Household(:,5);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 12 dummy variables

D1 = (M == 1); D2 = (M == 2);

D3 = (M == 3); D4 = (M == 4);

D5 = (M == 5); D6 = (M == 6);

D7 = (M == 7); D8 = (M == 8);

D9 = (M == 9); D10 = (M == 10);

D11 = (M == 11); D12 = (M == 12);

X=[a t D12 x];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

T0 = 50;

h = 3; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

D5t = D5(1:t); D6t = D6(1:t);

D7t = D7(1:t); D8t = D8(1:t);

D9t = D9(1:t); D10t = D10(1:t);

D11t = D11(1:t); D12t = D12(1:t);

xt = x(1:t);

Xt = [ ones(t,1) (1:t)' D12t xt];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [1 t+h D12(t+h) x(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

b=1:461;

m=53:461;

plot(b,y);

hold on

plot(m,syhat);

hold off

**PJS4:**

load 'Household.csv';

y = Household(:,2); M = Household (:,3);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 12 dummy variables

D1 = (M == 1); D2 = (M == 2);

D3 = (M == 3); D4 = (M == 4);

D5 = (M == 5); D6 = (M == 6);

D7 = (M == 7); D8 = (M == 8);

D9 = (M == 9); D10 = (M == 10);

D11 = (M == 11); D12 = (M == 12);

X=[t D1 D2 D3 D4 D5 D6 D7 D8 D9 D10 D11 D12];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

MSE=mean((y-yhat).^2);

AIC=T\*MSE+13\*2;

BIC=T\*MSE+13\*log(T);

T0 = 50;

h = 1; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

D5t = D5(1:t); D6t = D6(1:t);

D7t = D7(1:t); D8t = D8(1:t);

D9t = D9(1:t); D10t = D10(1:t);

D11t = D11(1:t); D12t = D12(1:t);

Xt = [(1:t)' D1t D2t D3t D4t D5t D6t D7t D8t D9t D10t D11t D12t];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [t+h D1(t+h) D2(t+h) D3(t+h) D4(t+h) D5(t+h) D6(t+h) D7(t+h) D8(t+h) D9(t+h) D10(t+h) D11(t+h) D12(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

H = 3; % h-step-ahead forecast

syHat = zeros(T-H-T0+1,1);

ytpH = y(T0+H:end); % observed y\_{t+h}

for t = T0:T-H

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

D5t = D5(1:t); D6t = D6(1:t);

D7t = D7(1:t); D8t = D8(1:t);

D9t = D9(1:t); D10t = D10(1:t);

D11t = D11(1:t); D12t = D12(1:t);

Xt = [(1:t)' D1t D2t D3t D4t D5t D6t D7t D8t D9t D10t D11t D12t];

beta2 = (Xt'\*Xt)\(Xt'\*yt);

yhat2 = [t+H D1(t+H) D2(t+H) D3(t+H) D4(t+H) D5(t+H) D6(t+H) D7(t+H) D8(t+H) D9(t+H) D10(t+H) D11(t+H) D12(t+H)]\*beta2;

syHat(t-T0+1) = yhat2;

end

MSFE2 = mean((ytpH-syHat).^2);

b=1:461;

c=51:461;

plot(b,y);

hold on

plot(c,syhat);

hold off

m=53:461;

plot(b,y);

hold on

plot(m,syHat);

hold off

**PJS4M:**

load 'Household.csv';

y = Household(:,2); M = Household (:,3);x=Household(:,5);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 12 dummy variables

D1 = (M == 1); D2 = (M == 2);

D3 = (M == 3); D4 = (M == 4);

D5 = (M == 5); D6 = (M == 6);

D7 = (M == 7); D8 = (M == 8);

D9 = (M == 9); D10 = (M == 10);

D11 = (M == 11); D12 = (M == 12);

X=[t D1 D2 D3 D4 D5 D6 D7 D8 D9 D10 D11 D12 x];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

T0 = 50;

h = 1; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

D5t = D5(1:t); D6t = D6(1:t);

D7t = D7(1:t); D8t = D8(1:t);

D9t = D9(1:t); D10t = D10(1:t);

D11t = D11(1:t); D12t = D12(1:t);

xt = x(1:t);

Xt = [(1:t)' D1t D2t D3t D4t D5t D6t D7t D8t D9t D10t D11t D12t xt];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [t+h D1(t+h) D2(t+h) D3(t+h) D4(t+h) D5(t+h) D6(t+h) D7(t+h) D8(t+h) D9(t+h) D10(t+h) D11(t+h) D12(t+h) x(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE2 = mean((ytph-syhat).^2);

b=1:461;

m=51:461;

plot(b,y);

hold on

plot(m,syhat);

hold off

ss=(1/(T-14))\*sum((ytph-syhat).^2);

fu=yhat1 +1.96\*sqrt(ss);

fd=yhat1 -1.96\*sqrt(ss);

u=mean(ytph);

Z=(110-u)/sqrt(ss);

**PJS4M3:**

load 'Household.csv';

y = Household(:,2); M = Household (:,3);x=Household(:,5);

T = length(y); t = (1:T)';a=ones(461,1);

%% construct 12 dummy variables

D1 = (M == 1); D2 = (M == 2);

D3 = (M == 3); D4 = (M == 4);

D5 = (M == 5); D6 = (M == 6);

D7 = (M == 7); D8 = (M == 8);

D9 = (M == 9); D10 = (M == 10);

D11 = (M == 11); D12 = (M == 12);

X=[t D1 D2 D3 D4 D5 D6 D7 D8 D9 D10 D11 D12 x];

betahat=(X'\*X)\(X'\*y);

yhat=X\*betahat;

T0 = 50;

h = 3; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

D1t = D1(1:t); D2t = D2(1:t);

D3t = D3(1:t); D4t = D4(1:t);

D5t = D5(1:t); D6t = D6(1:t);

D7t = D7(1:t); D8t = D8(1:t);

D9t = D9(1:t); D10t = D10(1:t);

D11t = D11(1:t); D12t = D12(1:t);

xt = x(1:t);

Xt = [(1:t)' D1t D2t D3t D4t D5t D6t D7t D8t D9t D10t D11t D12t xt];

beta1 = (Xt'\*Xt)\(Xt'\*yt);

yhat1 = [t+h D1(t+h) D2(t+h) D3(t+h) D4(t+h) D5(t+h) D6(t+h) D7(t+h) D8(t+h) D9(t+h) D10(t+h) D11(t+h) D12(t+h) x(t+h)]\*beta1;

syhat(t-T0+1) = yhat1;

end

MSFE2 = mean((ytph-syhat).^2);

b=1:461;

m=53:461;

plot(b,y);

hold on

plot(m,syhat);

hold off

**FminsearchforHolt:**

tic;

load 'Household.csv';

y = Household(:,2);y = log(y);

HouseholdHoltFunction([0.7 0.3 0.5]);%MSFE for aplha = 0.7, beta = 0.3, gamma = 0.5

[parametersstar MSFEstar] = fminsearch(@HouseholdHoltFunction, [0.2 0.2 0.2])

toc

function MSFE = HouseholdHoltFunction(parameters)

load 'Household.csv';

y = Household(:,2); T = length(y);

T0 = 50; h = 3; s = 12;

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

St = zeros(T-h,1);

alpha = parameters(1); beta = parameters(2); gamma = parameters(3);% input smoothing parameters

Lt = mean(y(1:s)); bt = 0; St(1:12) = y(1:s) - Lt;

for t = s+1:T-h

newLt = alpha\*(y(t) - St(t-s)) + (1-alpha)\*(Lt+bt);

newbt = beta\*(newLt-Lt) + (1-beta)\*bt;

St(t) = gamma\*(y(t)-Lt - bt) + (1-gamma)\*St(t-s);

yhat = newLt + h\*newbt + St(t+h-s);

Lt = newLt; bt = newbt; % update Lt and bt

if t>= T0

syhat(t-T0+1,:) = yhat;

end

end

syhat = exp(syhat); ytph = exp(ytph);

MSFE = mean((ytph-syhat).^2);

end

**Holt-winter**

load 'Household.csv';

y = Household(:,2);

T = length(y); t = (1:T)';a=ones(461,1);

T0 = 50;

H = 1; % h-step-ahead forecast

s = 12;

syHat = zeros(T-H-T0+1,1);

ytpH = y(T0+H:end); % observed y\_{t+h}

alpha = 0.666; beta = 0.0589; gamma = 0.9215;

St = zeros(T-H,1);

Lt = mean(y(1:s)); bt = 0; St(1:s) = y(1:s)-Lt;

for t= s+1:T-H

newLt = alpha\*(y(t)-St(t-s))+(1-alpha)\*(Lt+bt);

newbt = beta\*(newLt-Lt)+(1-beta)\*bt;

St(t) = gamma\*(y(t)-newLt)+(1-gamma)\*St(t-s);

yhat = newLt +H\*newbt + St(t+H-s);

Lt = newLt; bt = newbt;

if t>=T0

syHat(t-T0+1,:) = yhat;

end

end

MSFE3 = mean((ytpH - syHat).^2);

ss=(1/(T-3))\*sum((ytpH-syHat).^2);

fu=yhat +1.96\*sqrt(ss);

fd=yhat -1.96\*sqrt(ss);

u=mean(ytpH);

Z=(110-u)/sqrt(ss);

**Trend cycle lc:**

function loglike = trend\_cycle\_lc(omega,y)

T = length(y);

t = (1:T)'; %% build a vector from 1 to T

X = [ones(T,1) t cos(omega\*t) sin(omega\*t)];

beta = (X'\*X)\(X'\*y);

err = y-X\*beta;

sig2 = err'\*err/T;

loglike = -(-T/2\*log(2\*pi\*sig2) -.5\*(err'\*err)/sig2);

**Trend cycle:**

load 'Household.csv';

y = Household(:,2);T = length(y);

f = @(w) trend\_cycle\_lc(w,y); %% define f(w)

%% find the min of f from 2\*pi/40 to 2\*pi/4

omega = fminbnd(f,2\*pi/40,2\*pi/4);

t = (1:T)';

X = [ones(T,1) t cos(omega\*t) sin(omega\*t)];

beta = (X'\*X)\(X'\*y);

err = y-X\*beta;

sig2 = err'\*err/T;

T0 = 50;

h = 1; % h-step-ahead forecast

syhat = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y\_{t+h}

for t = T0:T-h

yt = y(1:t);

yhat1 = [1 t+h cos(omega\*(t+h)) sin(omega\*(t+h))]\*beta;

syhat(t-T0+1) = yhat1;

end

MSFE1 = mean((ytph-syhat).^2);

**AR (1):**

load 'Household.csv';

y = Household(:,2);

m = 2; %% the first m points as lags

y0 = y(1:m); y = y(m+1:end); T = length(y);

T0 = 50; h = 1; % h?step?ahead forecast

yhatAR = zeros(T-h-T0+1,1); %% AR(1) forecasts

for t = T0:T-h

yt = y(h:t);

zt = [ones(t-h+1,1) [y0(m);y(1:t-h)]];

betahat1=(zt'\*zt)\(zt'\*yt);

%% store the forecasts

yhatAR(t-T0+1,:) = [1 y(t)]\*betahat1;

end

ytph = y(T0+h:end); % observed y {t+h}

MSFE\_AR1 = mean((ytph-yhatAR).^2);

**AR (2):**

load 'Household.csv';

y = Household(:,2);

m = 3; %% the first m points as lags

y0 = y(1:m); y = y(m+1:end); T = length(y);

T0 = 50; h = 1; % h?step?ahead forecast

yhatAR = zeros(T-h-T0+1,1); %% AR(1) forecasts

for t = T0:T-h

yt = y(h:t);

zt = [ones(t-h+1,1) [y0(m);y(1:t-h)] [y0(m-1:end);y(1:t-h-1)]];

betahat2=(zt'\*zt)\(zt'\*yt);

%% store the forecasts

yhatAR(t-T0+1,:) = [1 y(t) y(t-1)]\*betahat2;

end

ytph = y(T0+h:end); % observed y {t+h}

MSFE\_AR2 = mean((ytph-yhatAR).^2);

**AR (3):**

load 'Household.csv';

y = Household(:,2);

m = 4; %% the first m points as lags

y0 = y(1:m); y = y(m+1:end); T = length(y);

T0 = 50; h = 1; % h?step?ahead forecast

yhatAR = zeros(T-h-T0+1,1); %% AR(1) forecasts

for t = T0:T-h

yt = y(h:t);

zt = [ones(t-h+1,1) [y0(m);y(1:t-h)] [y0(m-1:end);y(1:t-h-1)] [y0(m-2:end);y(1:t-h-2)]];

betahat2=(zt'\*zt)\(zt'\*yt);

%% store the forecasts

yhatAR(t-T0+1,:) = [1 y(t) y(t-1) y(t-2) ]\*betahat2;

end

ytph = y(T0+h:end); % observed y {t+h}

MSFE\_AR3 = mean((ytph-yhatAR).^2);

**AR (4):**

load 'Household.csv';

y = Household(:,2);

m = 5; %% the first m points as lags

y0 = y(1:m); y = y(m+1:end); T = length(y);

T0 = 50; h = 1; % h?step?ahead forecast

yhatAR = zeros(T-h-T0+1,1); %% AR(1) forecasts

for t = T0:T-h

yt = y(h:t);

zt = [ones(t-h+1,1) [y0(m);y(1:t-h)] [y0(m-1:end);y(1:t-h-1)] [y0(m-2:end);y(1:t-h-2)] [y0(m-3:end);y(1:t-h-3)]];

betahat2=(zt'\*zt)\(zt'\*yt);

%% store the forecasts

yhatAR(t-T0+1,:) = [1 y(t) y(t-1) y(t-2) y(t-3)]\*betahat2;

end

ytph = y(T0+h:end); % observed y {t+h}

MSFE\_AR4 = mean((ytph-yhatAR).^2);

**Loglike\_ARMA (1,1):**

%% input: x = [phi1,mu, psi];

%% input: y = data; y0 = lags;

function ell = loglike\_ARMA11(x,y0,y)

phi = zeros(2,1);

phi(1) = x(1); phi(2) = x(2);

psi = x(3); %phi(2) is mu

N = length(y); m = length(y0);

A = speye(N); B = sparse(2:N,1:N-1,ones(1,N-1),N,N);

Gam = A + B\*psi; Gam2 = Gam\*Gam';

X = [[y0(m);y(1:N-1)] ones(N,1)];

ell = -(y-X\*phi)'\*(Gam2\(y-X\*phi));

ell = - ell;

**Loglike\_ARMA (2,1):**

%% input: x = [phi1, phi2, mu, psi];

%% input: y = data; y0 = lags;

function ell = loglike\_ARMA21(x,y0,y)

phi = zeros(3,1);

phi(1) = x(1); phi(2) = x(2);

phi(3) = x(3); psi = x(4); %phi(3) is mu

N = length(y); m = length(y0);

A = speye(N); B = sparse(2:N,1:N-1,ones(1,N-1),N,N);

Gam = A + B\*psi; Gam2 = Gam\*Gam';

X = [[y0(m);y(1:N-1)] [y0(m-1:end);y(1:N-2)] ones(N,1)];

ell = -(y-X\*phi)'\*(Gam2\(y-X\*phi));

ell = - ell;

**Loglike\_ARMA (3,1):**

%% input: x = [phi1, phi2,phi3,mu, psi];

%% input: y = data; y0 = lags;

function ell = loglike\_ARMA31(x,y0,y)

phi = zeros(4,1);

phi(1) = x(1); phi(2) = x(2);

phi(3) = x(3);phi(4)=x(4); psi = x(5); %phi(4) is mu

N = length(y); m = length(y0);

A = speye(N); B = sparse(2:N,1:N-1,ones(1,N-1),N,N);

Gam = A + B\*psi; Gam2 = Gam\*Gam';

X = [[y0(m);y(1:N-1)] [y0(m-1:end);y(1:N-2)] [y0(m-2:end);y(1:N-3)] ones(N,1)];

ell = -(y-X\*phi)'\*(Gam2\(y-X\*phi));

ell = - ell;

**Loglike\_ARMA (4,1):**

%% input: x = [phi1, phi2, phi3, phi4, mu, psi];

%% input: y = data; y0 = lags;

function ell = loglike\_ARMA41(x,y0,y)

phi = zeros(5,1);

phi(1) = x(1); phi(2) = x(2);

phi(3) = x(3);phi(4)=x(4); phi(5)=x(5);psi = x(6); %phi(4) is mu

N = length(y); m = length(y0);

A = speye(N); B = sparse(2:N,1:N-1,ones(1,N-1),N,N);

Gam = A + B\*psi; Gam2 = Gam\*Gam';

X = [[y0(m);y(1:N-1)] [y0(m-1:end);y(1:N-2)] [y0(m-2:end);y(1:N-3)] [y0(m-3:end);y(1:N-4)] ones(N,1)];

ell = -(y-X\*phi)'\*(Gam2\(y-X\*phi));

ell = - ell;

**ARMA (1,1):**

%%ARMA11

m = 4; %% the first m points as lags

load 'Household.csv';

y = Household(:,2);

y0 = y(1:m); y = y(m+1:end); T = length(y);

T0 = 50; h = 1;

yhatMA = zeros(T-h-T0+1,1); %% ARMA(1,1) forecasts

%theta = [phi 1,mu, psi]

f = @(theta) loglike\_ARMA11(theta,y0,y(1:T0));

thetahat = fminsearch(f,[.5;0;.5]);

for t = T0:T-h

yt = y(1:t);

%% find the MLE

f = @(theta) loglike\_ARMA11(theta,y0,yt);

thetahat = fminsearch(f, thetahat);

%% make uhat

H = speye(t) + sparse(2:t,1:t-1,ones(1,t-1),t,t)\*thetahat(3);

X = [[y0(m);y(1:t-1)] ones(t,1)];

uhat = H\(yt - X\*[thetahat(1) thetahat(2)]');

%% store the forecasts

yhatMA(t-T0+1,:) = thetahat(2) + thetahat(1)\*y(t) + thetahat(3)\*uhat(end);

end

ytph = y(T0+h:end); % observed y {t+h}

MSFEMA = mean((ytph-yhatMA).^2);

**ARMA (2,1):**

%%ARMA21

m = 4; %% the first m points as lags

load 'Household.csv';

y = Household(:,2);

y0 = y(1:m); y = y(m+1:end); T = length(y);

T0 = 50; h = 1;

yhatMA = zeros(T-h-T0+1,1); %% ARMA(2,1) forecasts

%theta = [phi 1, phi 2, mu, psi]

f = @(theta) loglike\_ARMA21(theta,y0,y(1:T0));

thetahat = fminsearch(f,[.5;.5;0;.5]);

for t = T0:T-h

yt = y(1:t);

%% find the MLE

f = @(theta) loglike\_ARMA21(theta,y0,yt);

thetahat = fminsearch(f, thetahat);

%% make uhat

H = speye(t) + sparse(2:t,1:t-1,ones(1,t-1),t,t)\*thetahat(4);

X = [[y0(m);y(1:t-1)] [y0(m-1:end);y(1:t-2)] ones(t,1)];

uhat = H\(yt - X\*[thetahat(1) thetahat(2) thetahat(3)]');

%% store the forecasts

yhatMA(t-T0+1,:) = thetahat(3) + thetahat(1)\*y(t)+ thetahat(2)\*y(t-1) + thetahat(4)\*uhat(end);

end

ytph = y(T0+h:end); % observed y {t+h}

MSFEMA = mean((ytph-yhatMA).^2);

**ARMA (3,1):**

%%ARMA31

m = 4; %% the first m points as lags

load 'Household.csv';

y = Household(:,2);

y0 = y(1:m); y = y(m+1:end); T = length(y);

T0 = 50; h = 1;

yhatMA = zeros(T-h-T0+1,1); %% ARMA(3,1) forecasts

%theta = [phi 1, phi 2, phi 3; mu, psi]

f = @(theta) loglike\_ARMA21(theta,y0,y(1:T0));

thetahat = fminsearch(f,[.5;.5;0.5;0;5]);

for t = T0:T-h

yt = y(1:t);

%% find the MLE

f = @(theta) loglike\_ARMA21(theta,y0,yt);

thetahat = fminsearch(f, thetahat);

%% make uhat

H = speye(t) + sparse(2:t,1:t-1,ones(1,t-1),t,t)\*thetahat(5);

X = [[y0(m);y(1:t-1)] [y0(m-1:end);y(1:t-2)] [y0(m-2:end);y(1:t-3)] ones(t,1)];

uhat = H\(yt - X\*[thetahat(1) thetahat(2) thetahat(3) thetahat(4)]');

%% store the forecasts

yhatMA(t-T0+1,:) = thetahat(4) + thetahat(1)\*y(t)+ thetahat(2)\*y(t-1)++ thetahat(3)\*y(t-2) + thetahat(5)\*uhat(end);

end

ytph = y(T0+h:end); % observed y {t+h}

MSFE = mean((ytph-yhatMA).^2);

**ARMA (4,1):**

%%ARMA41

m = 4; %% the first m points as lags

load 'Household.csv';

y = Household(:,2);

y0 = y(1:m); y = y(m+1:end); T = length(y);

T0 = 50; h = 1;

yhatMA = zeros(T-h-T0+1,1); %% ARMA(4,1) forecasts

%theta = [phi 1, phi 2, phi 3; mu, psi]

f = @(theta) loglike\_ARMA41(theta,y0,y(1:T0));

thetahat = fminsearch(f,[.5;.5;.5;.5;0;.5]);

for t = T0:T-h

yt = y(1:t);

%% find the MLE

f = @(theta) loglike\_ARMA41(theta,y0,yt);

thetahat = fminsearch(f, thetahat);

%% make uhat

H = speye(t) + sparse(2:t,1:t-1,ones(1,t-1),t,t)\*thetahat(5);

X = [[y0(m);y(1:t-1)] [y0(m-1:end);y(1:t-2)] [y0(m-2:end);y(1:t-3)] [y0(m-3:end);y(1:t-4)] ones(t,1)];

uhat = H\(yt - X\*[thetahat(1) thetahat(2) thetahat(3) thetahat(4) thetahat(5)]');

%% store the forecasts

yhatMA(t-T0+1,:) = thetahat(5) + thetahat(1)\*y(t)+ thetahat(2)\*y(t-1)+ thetahat(3)\*y(t-2)+ thetahat(4)\*y(t-3) + thetahat(6)\*uhat(end);

end

ytph = y(T0+h:end); % observed y {t+h}

MSFE = mean((ytph-yhatMA).^2);

**IAR1:**

m = 4; %% the first m points as lags

load 'Household.csv';

y = Household(:,2);T0 = 50; h = 1; % h?step?ahead forecast

dely = y(m+h:end) - y(m:end-h);

y0 = y(1:m); y = y(m+1:end); T = length(y);

yhatIAR = zeros(T-h-T0+1,1); %% IMA(1,1) forecasts

ytph = y(T0+h:end); % observed y {t+h}

for t = T0:T-h

yt = y(h:t);

zt = [ones(t-h+1,1) [y0(m);y(1:t-h)]];

betahat1=(zt'\*zt)\(zt'\*yt);

%% store the forecasts

yhatIAR(t-T0+1,:) = [1 y(t)]\*betahat1;

end

ytph = y(T0+h:end); % observed y {t+h}

MSFE\_IAR1 = mean((ytph-yhatIAR).^2);

**IAR2:**

m = 4; %% the first m points as lags

load 'Household.csv';

y = Household(:,2);T0 = 50; h = 1; % h?step?ahead forecast

dely = y(m+h:end) - y(m:end-h);

y0 = y(1:m); y = y(m+1:end); T = length(y);

yhatIAR = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y {t+h}

for t = T0:T-h

yt = y(h:t);

zt = [ones(t-h+1,1) [y0(m);y(1:t-h)] [y0(m-1:end);y(1:t-h-1)]];

betahat2=(zt'\*zt)\(zt'\*yt);

%% store the forecasts

yhatIAR(t-T0+1,:) = [1 y(t) y(t-1)]\*betahat2;

end

ytph = y(T0+h:end); % observed y {t+h}

MSFE\_IAR2 = mean((ytph-yhatIAR).^2);

**IAR3:**

m = 4; %% the first m points as lags

load 'Household.csv';

y = Household(:,2);T0 = 50; h = 1; % h?step?ahead forecast

dely = y(m+h:end) - y(m:end-h);

y0 = y(1:m); y = y(m+1:end); T = length(y);

yhatIAR = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y {t+h}

for t = T0:T-h

yt = y(h:t);

zt = [ones(t-h+1,1) [y0(m);y(1:t-h)] [y0(m-1:end);y(1:t-h-1)] [y0(m-2:end);y(1:t-h-2)]];

betahat2=(zt'\*zt)\(zt'\*yt);

%% store the forecasts

yhatIAR(t-T0+1,:) = [1 y(t) y(t-1) y(t-2) ]\*betahat2;

end

ytph = y(T0+h:end); % observed y {t+h}

MSFE\_IAR3 = mean((ytph-yhatIAR).^2);

**IAR4:**

m = 4; %% the first m points as lags

load 'Household.csv';

y = Household(:,2);T0 = 50; h = 1; % h?step?ahead forecast

dely = y(m+h:end) - y(m:end-h);

y0 = y(1:m); y = y(m+1:end); T = length(y);

yhatIAR = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y {t+h}

for t = T0:T-h

yt = y(h:t);

zt = [ones(t-h+1,1) [y0(m);y(1:t-h)] [y0(m-1:end);y(1:t-h-1)] [y0(m-2:end);y(1:t-h-2)] [y0(m-3:end);y(1:t-h-3)]];

betahat2=(zt'\*zt)\(zt'\*yt);

%% store the forecasts

yhatIAR(t-T0+1,:) = [1 y(t) y(t-1) y(t-2) y(t-3)]\*betahat2;

end

ytph = y(T0+h:end); % observed y {t+h}

MSFE\_IAR4 = mean((ytph-yhatIAR).^2);

**Loglike\_UC:**

function ell = loglike\_UC(sig,omega,y,Vtau)

T = length(y);

%% compute the MLE for tau given sig and omega

H = speye(T) - sparse(2:T,1:(T-1),ones(1,T-1),T,T);

invOmega = sparse(1:T,1:T,[1/Vtau 1/omega\*ones(1,T-1)]);

HinvOmegaH = H'\*invOmega\*H;

K = speye(T)/sig + HinvOmegaH;

tauhat = K\(y/sig);

err = (y-tauhat)'\*(y-tauhat)/sig + tauhat'\*HinvOmegaH\*tauhat;

ell = -T/2\*log(sig) - (T-1)/2\*log(omega) - .5\*err;

ell = -ell;

**UC model:**

load 'Household.csv';

y = Household(:,2);

m = 4; %% the first m points as lags

y0 = y(1:m); y = y(m+1:end);

T = length(y);

Vtau = 9; %% initial condition

omega = 1; %% fix omega

%% recursive forecast exercise

T0 = 50;

h = 1; % h?step?ahead forecast

yhatUC = zeros(T-h-T0+1,1);

ytph = y(T0+h:end); % observed y {t+h}

sighat = 1;

for t = T0:T-h

yt = y(1:t);

f = @(sig) loglike\_UC(sig,omega,yt,Vtau);

sighat = fminsearch(f,sighat);

H = speye(t) - spdiags(ones(t-1,1),-1,t,t);

invOmega =sparse(1:t,1:t,[1/Vtau 1/omega\*ones(1,t-1)],t,t);

HinvOmegaH = H'\*invOmega\*H;

K = speye(t)/sighat + HinvOmegaH;

tauhat = K\(yt/sighat);

yhatUC(t-T0+1) = tauhat(end); %store the forecasts

end

ytph = y(T0+h:end); % observed y {t+h}

MSFEuc = mean((ytph-yhatUC).^2);